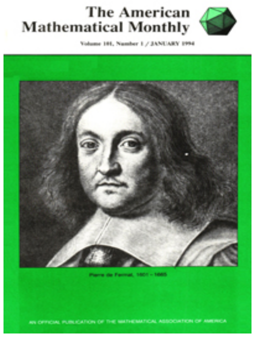

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Mathematical Cranks by Underwood Dudley

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REVIEWS

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Mathematical Cranks by Underwood Dudley. The Mathematical Association of America, Washington, 1992, v + 372, $25.00.

Reviewed by **Ian Stewart**

A few years ago I wrote a short article in *New Scientist*, in which I made passing reference to "the Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, \ldots$." (The article was actually about the fractal structure of diffusion-limited aggregates, but that's by the by.) Shortly afterwards the editors received a somewhat vituperative letter complaining about my appalling ignorance. I explained to the editors why I believed myself to be blameless, and they spiked the letter. Over the next few weeks they received repeated telephone calls from the same person, arguing the same point; he also tried to telephone me to discuss the matter, but my secretary managed to divert him with lies about my absence.

What was this terrible error that I had made, an error so important that the attempt to have it corrected occupied so much time and effort?

Simple. The Fibonacci series goes *zero*, $1, 1, 2, 3, 5, 8, 13, \ldots$.

I don't particularly want to argue the toss here, but I think most mathematicians would concur that whether you include an initial zero is largely a matter of convention. As it happened, I had excellent journalistic reasons for excluding it. The pattern of numbers then fitted the experimentalists' picture precisely. If I had included the zero, I would also have had to spend several sentences explaining why it didn't show up in the picture. All of this would be a distraction from the main point—about pattern-formation—and the word limit of 500 was already tight.

I don't *know* that the person involved was a crank—but they exhibited one of the crucial symptoms, an obsession with trivialities. Very *specific* trivialities: I imagine he specialised in Fibonacci numerology. He was probably a perfectly reasonable person on most matters, but push the Fibonacci button and he would perform.

Cranks are a pest: to their families and friends, to us professionals, and to themselves. So why would anybody write an entire book about them? Because, says Underwood Dudley, 'cranks are *interesting*,' And he's right.

Mathematical Cranks fits into a tiny but respected genre, which began with Augustus de Morgan's *A Budget of Paradoxes*. Martin Gardner's *Fads and Fallacies* is another, though not addressed to mathematics; and there's also Dudley's own *A Budget of Trisections*. Dudley collects cranks—it would be more prosaic, but less representative of the spirit of the enterprise, to say he collects crank *writings*—and he has solved a major problem for many of us. Before, when we received a crank letter, we would have to debate whether to answer it (and risk an endless and pointless ensuing correpondence), be exceedingly rude to its author in the hope that he (it virtually always is a *he*) will go away, or just bin it. (I must here confess to once binning a letter from a very nice doctor whose *patient* was the crank. The doctor had no way of knowing whether the patient's mathematical research was valuable or not, and wanted professional advice. My problem was that the patient was at the time confined to a secure unit for the mentally ill that specialises in homicidal maniacs. Would *you* tell such a person—however nice and well-meaning his doctor is—that his life's work on Fermat's Last Theorem is complete trash?) Be that as it may, Dudley's hobby of crank-collection has solved all such dilemmas. Now I just shove the lot in an envelope and send it to *him*. I recommend that you all do the same. (He's at DePauw University, Greencastle, IN. No, look, he really *wants* this stuff. ...)

The activities exposed in *Mathematical Cranks* take a variety of forms, ranging from utter nonsense to ideas that might—in other peoples' hands—form the basis of some genuine research projects. The Top Ten of mathematical crankhood is

1 Squaring the Circle
2 Trisecting the Angle
3 Fermat's Last Theorem
4 Proving the Parallel Postulate
5 The Golden Number
6 Perfect Numbers
7 The Four Colour Theorem
8 Number Bases
9 Cantor's Diagonal Argument
10 Duplicating the Cube.

It may seem strange that the 'three classical problems of antiquity' (actually nothing of the sort) come as numbers 1, 2, and then a huge gap until 10. But cranks' intuition agrees with that of most mathematicians: duplicating the cube is *boring* in comparison to the other two. Number base cranks advocate a change from base ten to something more sensible, usually 12 but sometimes 8 or 16. They are cranks not because this is a bad idea—on the contrary, base 12 would make excellent sense—but because as a matter of practicality it is even less likely to come about than a change from the QWERTY keyboard. (My own view here is that it would be simpler to reorder the alphabet to start at *Q*. Let me tell you all about it, it shouldn't take more than a week...) The four-colour theorem has dropped down the charts since Appel-Haken: nowadays cranks confine themselves to seeking simple proofs—or at least proofs simpler than many hours of supercomputer time. Cantor's proof of the uncountability of the real numbers is perhaps the most sophisticated member of the Top Ten; Fermat's last theorem is the most significant open problem (added in proof: well, it <u>was</u>); and the parallel postulate has had the greatest influence on mathematics.

There are three patterns to this list. The first is that cranks have an unbounded belief in their own ability to solve problems for which there is a theorem that no solution exists. The second is that if a result is counter-intuitive, a crank will prefer to deny it rather than to refine his intuition. The third is that the kind of problem that cranks know about is the sort of thing that a schoolteacher might throw out to the odd bright pupil. While the rest of the class is struggling with how to bisect angles, there's one little lad with shining eyes who's set his sights on dividing them into n equal parts. Kindly teacher points out that the case $n = 3$ is known to be impossible; or perhaps just vaguely recalls being told that nobody has ever been able to solve it. At that instant a crank is born: a message is imprinted somewhere in the depths the child's brain. That message is *not* that the problem has been understood long ago: it is that it remains unsolved and that vast fame and fortune await the intrepid solver. It is in the nature of the disease that it will then lie dormant for anything up to 50 years, to reveal its deadly symptoms only upon retirement, when the sufferer has finally found the time to work on the question that has been festering away these past many decades.

Teachers (and I here count myself as one): *do not worry about this*. We do far more good to the majority of our students by making it clear that mathematics includes both unsolved problems and unsolvable ones. We also do far more damage to our students in other unavoidable ways, and we usually never know it. That is in the nature of our chosen calling, just as doctors kill most of their patients in the long run. In any case, it seems likely that crankhood is in fact a genuine medical condition. There is a known disease of the mind—regrettably I can't recall its name—whose symptoms include an obsession with very specific trivia. Cranks surely suffer from some version of this.

Some samples.

There is the fascinating case of HJ (Dudley uses initials only on the grounds that it is the ideas of cranks, not the people themselves, that he wishes to discuss) who invented Celestial Calculus. Ordinary calculus adds a little bit (δx) to x and sees by how much (δy) the function $y = f(x)$ changes. HJ *multiplies* x by something a little bit bigger than 1 and see how much y multiplies by. This is a neat idea. It replaces the usual differential operator $D = d/dx$ by a different one L, which can be expressed as

$$L(y) = \frac{d(\ln y)}{d(\ln x)}.$$

It's rather cute:

$$L(x^n) = n$$

$$L(e^x) = x$$

$$L(uv^{\pm 1}) = L(u) \pm L(v).$$

The formula for $L(u + v)$ is not so cute, however, and $L(L(u))$ is a pig. A keen calculus class would have fun with this operator. What's so cranky here? Nothing —until you read HJ's estimation of its importance: "Clearly the most unifying and comprehensive identity in mathematics."

WD simply couldn't bear the Banach-Tarski Theorem, that a sphere can be cut into a finite number of pieces and reassembled to give two spheres the same size as the original. This would be paradoxical if it referred to physical spheres and to pieces that could actually be *made*, but it doesn't and it's not. WD realised that the proof of the Banach-Tarski Theorem depended on set theory, so he decided that set theory had to go. His chosen weak link was Cantor's diagonal proof that the real numbers are uncountable, and his idea was to construct a countable list of all real numbers. Buried in the verbiage is the list: .1, .2, .3, .4, .5, .6, .7, .8, .9, .10, .11, .12, .13,... and so on. Students often make the same error, but they usually understand when it is pointed out that the list includes only the terminating decimals. Cranks don't.

JB mailed a two-page paper to every math department in the United States, with the original and succinct title:

1

His discovery is the impressive-looking formula

$$\sum_{N^E}^{(N-1)^E} \left(E\sqrt{N^E} - \sqrt[E]{N^E - 1} \right) = 1.$$

The notation's slightly adrift, and it's a useful exercise to correct it. He meant that, for example, when $N = 3$, $E = 2$ the sum should be

$$(\sqrt{9} - \sqrt{8}) + (\sqrt{8} - \sqrt{7}) + (\sqrt{7} - \sqrt{6}) + (\sqrt{6} - \sqrt{5}) + (\sqrt{5} - \sqrt{4}).$$

Think about it.

GB, some kind of engineer in his mid-fifties, spent a lot of time trying to find good ways to calculate tables of primes and the like. As a mathematician who knew him explains: "He genuinely expected some sort of ovation from the mathematical community, which he envisioned squirming in its chairs for want of good methods of calculating square roots, tabling Pythagorean triples, and listing the primes... What he has been doing lately is this: he considers the one-sided infinite matrix whose (i, j) entry is $i^2 + j$, and looks at the numbers that lie on lines, parabolas, and hyperbolas drawn in this array, with the idea of discovering patterns. He has discovered some God-awful patterns so far." For instance, the diagonal running upwards to the right from entry (10, 1) reads 101, 83, 67, 53, 41, 31, 23, 17, 13, 11. All primes! One can see why GB was so interested. Unfortunately, because no polynomial can take only prime values, his search is ultimately doomed to failure.

Cranks have infinite confidence and zero modesty. LJ was so incensed about the primitive state of the theory of differential equations that he wrote *The Stupid, Ridiculous Oversight* to put the matter right. It is quite the exception among crank writing: it tackles such things as Yukawa's solution of the wave equation for the muon, and Schrödinger's equation for the energy levels of hydrogen. It contains an *exact* solution of van der Pol's equation, which LJ recasts as

$$y'' + Ay' + By^2y' + Cy = -Di\,e^{i\omega t}.$$

His solution takes the form

$$[\cos \omega t] \sum_{n=0}^{\infty} (\underline{n}/n!)(\omega t)^n.$$

We need to be told what $\underline{n}$ is; and LJ explains in a series of equations that went

$$\underline{0} = 1$$

$$\underline{1} = -(A + B)\omega^{-1}/2 - D\omega^{-2}/2$$

and so on until

$$\underline{4} = \left[A(3\,\underline{1} - \underline{3})\omega - B(\underline{3} - 27\,\underline{1} + 6\,\underline{12} + 2\,\underline{111})\omega - C(\underline{2} - 1) \right]\omega^{-2} + 6\underline{2} - 1, \ldots.$$

Dudley remarks that "the '...' doesn't help much, but never mind," and goes on to test the solution—whatever '...' means—by taking the extreme case $A = B = C = D = 0$. Now the equation becomes $y'' = 0$. The solution is

$$y = at + b$$

for constants a and b. LJ's solution, on the other hand, reduces to

$$y = (\cos \omega t)\left(1 + \frac{1}{2!}(\omega t)^2 + \frac{5}{4!}(\omega t)^4 + \dots\right).$$

"LJ must have committed an oversight, perhaps stupid and ridiculous."

As well as providing a representative survey of crank writings, and a generally compassionate but occasionally exasperated view of their authors, *Mathematical Cranks* also addresses various practical issues: how cranks are created, how to respond to them, and their (totally ungrounded) belief that it is possible to make money out of mathematics. In particular, we are urged *never* to encourage a crank with vague statements that *he* will read as praise but *you* know are the opposite—for example "If your work were to contain significant results, I would be happy to publish it in my journal." The crank doesn't see the 'if'—or if he does, he *knows* his results are significant. So why won't you publish them? Equally, it is highly dangerous to say "I found your results very interesting but I am not competent to judge them" when what you mean is "Your work is obvious trash but I'm not prepared to waste my time saying why". You may well find yourself quoted in the *next* version of the work—as being incompetent, but dazzled by its brilliance.

Mathematical Cranks is unique, and it exists. For both of these properties we should be grateful.

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Complex Analysis: The Geometric Viewpoint. By Steven G. Krantz. The Carus Mathematical Monographs, Number 23, The Mathematical Association of America, ix + 212, 1990.

Reviewed by **John Polking**

One of the more fruitful and beautiful approaches to complex function theory in one and in several variables involves the interaction between holomorphic functions and mappings and various natural Riemannian metrics. The applications of this geometric approach include value distribution theory. The simplest example of this theory is the Liouville Theorem which says that any bounded entire function must be a constant. A more difficult example is the Little Picard Theorem, which says that any entire holomorphic function whose range omits two points must be a constant. Another application of the geometric approach is the study of properties of the group of biholomorphisms of a domain. These can range from determination of the group in simple cases such as the disk or the plane, to finding conditions under which the group is compact or noncompact.

Historically the first element of the geometric approach was provided by the introduction of what is called the Poincaré metric on the unit disk by Poincaré in 1881. This is the Riemannian metric

$$ds^2 = \left(1 - |z|^2\right)^{-2}\left(dx^2 + dy^2\right).$$